D 26926 (Pages : 2) Name.....

Reg. No.....

FOURTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2006

EN 04 401-A—ENGINEERING MATHEMATICS—IV

(Common for all except CS and IT—2004 admissions)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Show that $\lim_{z \to 0} \frac{xy}{x^2 + y^2}$ does not exist.

(b) Define an analytic function $f(z)$.

(c) Find the residue of $f(z) = \frac{1}{(z^2 + 1)^2}$ about each singularity.

(d) Evaluate $\int \frac{z - 2}{z^2 - z} dz$ where C is the circle $x^2 + y^2 = 4$.

(e) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.

(f) Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

(g) Write down the possible solutions of one dimensional heat equation.

(h) Solve $2x \frac{\partial z}{\partial x} - 3y \frac{\partial z}{\partial y} = 0$ using method of separation of variables.

($8 \times 5 = 40$ marks)

II. (a) (i) Prove that the real and imaginary parts of an analytic function are solutions of Laplace equation in two dimensions. (8 marks)

(ii) Discuss the transformation $W = e^z$. (7 marks)

Or

(b) (i) Find the bilinear transformation which maps 0, 1, ∞ onto i, -1, $-i$. (8 marks)

(ii) Discuss the transformation $W = \frac{1}{z}$. (7 marks)

Turn over

III. (a) (i) Expand $f(z) = \dfrac{z^2 - 1}{(z + 2)(z + 3)}$ in a Laurents series if (i) $|z| < 2$; (ii) $|z| > 3$; and (ii) $2 < |z| < 3$.

(8 marks)

(ii) Evaluate $\int_0^{2\pi} \dfrac{d\theta}{2 + \cos\theta} = \dfrac{2\pi}{\sqrt{3}}$. (7 marks)

Or

(b) (i) Show that $\int_0^{2\pi} \dfrac{d\theta}{1 - 2a\cos\theta + a^2} = \dfrac{2\pi}{1 - a^2}$ if $|a| < 1$. (8 marks)

(ii) Evaluate $\int_C \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z - 1)^2 (z - 2)} dz$ where C is the circle $|z| = 3$. (7 marks)

IV. (a) Find the general solution in series of ascending powers of "x" of the differential equation

$$4x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = 0.$$

(15 marks)

Or

(b) Show that $y = \sqrt{x}$. Jn (ax) is a solution of $\dfrac{d^2y}{dx^2} + \left(a^2 - \dfrac{4n^2 - 1}{4x^2}\right) y = 0$. (15 marks)

V. (a) A rod 10 cm. with insulated lateral surface is initially at temperature $f(x)$ at an inner point distant 12 cm. from one end. If both the ends are kept at zero temperature, find the temperature at any point of the rod at any subsequent time.

(12 marks)

Or

(b) A square plate has its faces and the edge $y = 0$ insulated. Its edges $x = 0$ and $x = \pi$ are kept at zero temperature and its fourth edge $y = \pi$ is kept as temperature $f(x)$. Find the steady state temperature at any point of the plate.

(15 marks)